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Simulation of ECB decisions and forecast of short term Euro rate with an adaptive fuzzy expert system

Lorenzo Rizzi [a], Flavio Bazzana [a,*], Nikola Kasabov [b], Mario Fedrizzi [a], Luca Erzegovesi [a]

[a] *Department of Computer and Management Sciences, University of Trento, 38100 Trento, Italy*
[b] *Department of Information Science, University of Otago, New Zealand*

Abstract

The elaboration of optimal monetary policy strategies, and the statistical estimation of monetary policy rules followed by European Central Bank (ECB) in the new currency area of the Euro, are difficult to follow with the standard statistical models. For this reason we have developed an adaptive fuzzy expert system in order to mimic the framework on which the monetary policy strategy of the ECB is based. The expert system knowledge base consists of a set of fuzzy and crisp rules located at two different hierarchical levels. The high-level of the system receives some intermediate output values from the low-level and processes this information by means of a set of crisp rules. The low-level prepares these intermediate output values with the use of a fuzzy inference engine applied to economic input variables. The use of an expert system allows for modelling the ECB behaviour with the use of wider scope of knowledge, when compared with more traditional computational techniques. Rules at different hierarchical levels and at different intra-level groups, allow for managing the potentially contradictory structure of the ECB strategy. The system has been tested on the economic and financial time series going from the January 1999 to September 2000. The system's correct prediction was estimated to overall 70% and, considering the complexity of the task, the results obtained are promising.

Keywords: Monetary policy; European central bank; Expert systems; Fuzzy sets

1. Introduction

In the last decade, the attention dedicated in the academic field to the problem of monetary policy making has been remarkable. Pointers of this tendency are the amount of recent publications and conferences held on the argument, and the efforts produced by many researchers in order to define behavioural rules for central banks. The proposal by John Taylor of a simple rule based on the interest rate (Taylor, 1993) represents a well-known example as well as the research field based on inflation target (Svensson, 1998).

The literature on monetary policy has focused mainly on two directions. The first one is the elaboration of optimal monetary policy strategies

* Corresponding author. Address: DISA, Università di Trento, Via Inama n. 1, 38100 Trento, Italy. Tel.: +39-0461-883107; fax: +39-0461-882124.
E-mail address: bazzana@cs.unitn.it (F. Bazzana).

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given a certain structure of the economy, a set of rules governing it and an objective function by the monetary authorities (Clarida et al., 1999; Orphanides and Wieland, 2000; Svensson, 1997; Svensson, 1998). The second direction is the statistical estimation of monetary policy rules followed by the central banks on the base of past economic data, and their use as benchmark for the monetary policy (Bernake and Mihov, 1997; Clarida et al., 1998; Gerlach and Schnabel, 1999; Gerlach and Smets, 1998; Smets, 1998; Taylor, 1993; Taylor, 1999).

In the case of the new currency area of the Euro both the directions are, at least at the moment, difficult to follow. As regards the first one, the task is extremely difficult due to the high degree of uncertainty that surrounds the economic structure of the Euro area and the basic relations that govern its functioning. The shift to the common currency has caused important changes in the behaviour of economic actors, partially yet unknown, and the Euro area economic context is continuously changing.

As regards the second one, the main obstacle is the relatively short history of the monetary policy by the ECB. As known, the ECB has been in charge of the monetary policy since the 1st January 1999. About two years of data, most of them quarterly, are not enough for the statistical techniques commonly used for the estimation of the reaction functions.

Therefore, there is the need of a model that does not require the specification of the detailed functioning of the economy and does not rely on past economic and financial time series. In the present work, we have chosen a fuzzy expert system. Such a system, trying to mimic only the framework on which the monetary policy strategy of the ECB is based, does not require any further assumption on the structure of the economy. Besides, the expert system being fed with an initial knowledge through two different sets of fuzzy and crisp rules does not rely on any past time series.

In the last months of the 1998 the Governing Council of the European Central Bank has announced the principal elements of its monetary policy strategy (European Central Bank, 1999). This strategy is oriented towards maintaining price stability in the Euro area, following the mandate set in the Treaty that establishes the European Community.

The Eurosystem monetary policy is constituted by three main elements: a quantitative definition of the primary object, price stability, and two "pillars" on which the monetary policy strategy is based to achieve the primary object. The Maastricht Treaty, assigns the primary object of price stability to an independent central bank, [1] assuming that price stability is the best contribution that the monetary policy can give to the economic development in the Euro area.

In this context the Governing Council of the ECB has announced the following definition: "the price stability will be defined as an increase on the 12 months harmonised consumer price index lower than 2%". This definition sets the upper threshold that defines price stability and in the same time, using the word "increase", assumes that also a deflation is not supposed to be considered compatible with the definition of price stability. According to the definition given by the Eurosystem the price stability has to be achieved on a medium-term horizon. This reflects the evidence that the short-term evolution of the prices is influenced by factors on which the monetary policy has very little or even no control.

The first pillar assigns a prominent rule to money and is based on the announcement of a reference value for monetary growth. In this logical framework the Governing Council of the ECB, has fixed the reference value for the three months moving average of the 12 months growth rate of M3 [2] at 4.5%. Deviations of the monetary growth from this value require a careful analysis of the economic climate in order to verify the presence of risk for medium-term price stability.

Let us consider now the main features of the second pillar of the monetary policy strategy. The

[1] For different analysis regarding the problem of the "inflationary bias" see Blinder (1998).

[2] M3 is a monetary aggregate defined as the sum of money, bank deposits, and bonds with life less than six months. There are other two monetary aggregates: M2 defined as M3 minus bonds with life under six months, and M1 defined as M2 minus bank deposits, i.e. M1 is money in a strict sense.

presence of this second pillar is due to the consideration that the monetary indicators, even if they contain crucial information for the evolution of prices, are not enough for an in-depth and exhaustive analysis of the economic evolution. Therefore, in parallel with the analysis of monetary indicators, the Eurosystem monetary strategy provide for a wide evaluation of the perspective regarding the price evolution in the Euro area and the risks for their stability.

This evaluation takes into consideration a wide range of financial and economic indicators as well as some surveys among the private sector of the economy. These indicators should make clearer the interaction between demand and offer and the dynamics of costs in labour and goods market. Besides, in this part of the strategy the Eurosystem pays attention to the inflation forecasts published by the main international organisations as well as to their internal forecasts.

The second section will introduce the fuzzy sets based method used in the construction of the fuzzy expert systems, which represent the main result of our research. Such an expert system would be, in the intention of the authors, a decision support system for the forecast of intervention by the Eurosystem. The third section will present and comment the results of the empirical conclusions and suggestions for further research analysis, while the last section contains conclusions.

2. A fuzzy expert system modelling the decision process behind ECB monetary policy interventions

In the previous sections we have shown that the analysis of the inflationary picture should be ever done (considering the high degree of uncertainty surrounding the nowadays economy evolution) in the context of the two pillars approach. Anyway they should have also made clear that it is very difficult, if not impossible, to merge in a transparent way these two kinds of analysis in only one analytic approach. The standard statistical models commonly used for the modelling of monetary policy could present many problems in the logical representation of the two pillars complementarity. Anyway the ECB behaviour needs to be interpreted. For these reasons we have decided to use in the present work a fuzzy expert system which allows to deal with imprecise and vague knowledge (Fig. 1).

2.1. Fuzzy sets and fuzzy expert systems

Fuzzy sets were introduced by Zadeh in the 60s (Zadeh, 1965) in order to mathematically represent uncertainty and vagueness and to provide formalized tools for dealing with the imprecision intrinsic to many problems. The main idea behind fuzzy sets is the revision of the usual definition of the characteristic function of a set, so as to

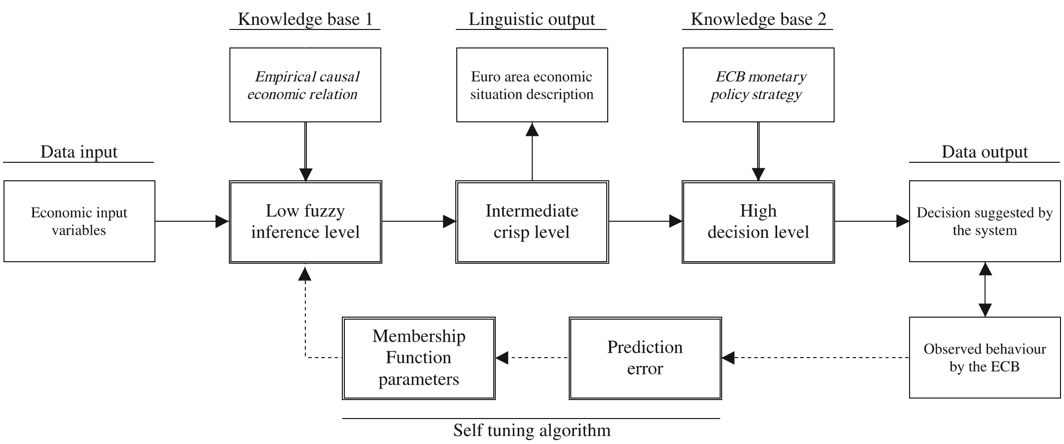

Fig. 1. ECB intelligent decision support system structure.

introduce degrees of membership. In classical set theory, a subset A of a set X can be defined by its characteristic function:

$$X_A : X \to \{0, 1\}.$$

The value zero is used to represent non-membership, and the value one is used to represent membership.

Similarly, a fuzzy subset of a set X can be defined by its membership function μ_A, a mapping from the elements of X to the elements of the interval [0,1]:

$$\mu_A : X \to [0, 1].$$

Fuzzy sets theory provides a host of mathematical tools for representing and aggregating the uncertain information in decision-making.

In 1979 Zadeh introduced the theory of approximate reasoning (Zadeh, 1979) to provide a framework for reasoning in the face of imprecise and uncertain knowledge. Central to this theory is the representation of propositions as statements assigning fuzzy sets as values to variables.

It is well known that traditional expert systems represent knowledge through a set of rules, which usually link a group of input variables to a certain output variable. When the knowledge is imprecise and uncertain it is necessary to develop adequate reasoning methods (Dubois and Prade, 1988). One way is to use fuzzy production rules, where the conditional part and/or the conclusion part contain linguistic variables, and the inference engine is based on approximate reasoning techniques (Fuller, 1998).

A linguistic variable is characterized by a quintuple (x, $T(x)$, U, G, M) in which x is the name of the variable, $T(x)$ is the set of names of the linguistic values, U is the universe of discourse, G is a syntax for generating the names of the value of x, and M is a semantic rule for associating with each value its meaning.

For example, if "economic growth" is interpreted as a linguistic variable, then its term set T could be, $T = \{\text{very slow, slow, moderate, more or less fast, fast}, \ldots\}$, and each term in T is represented by a fuzzy set in the universe of discourse U.

Usually the terms are represented by specific fuzzy sets called fuzzy numbers (Dubois and Prade, 1980). A fuzzy number N is a fuzzy subset of $X (\subseteq \Re)$ satisfying the following conditions:

(i) $\exists x \in \Re$ s.t. $\mu_N(x) = 1$;
(ii) $[N]^\alpha$ a convex subset of X $\forall\ \alpha \in [0, 1]$,

where $[N]^\alpha$ is called α-cut and defined as:

$$[N]^\alpha = \begin{cases} x \in X | \mu_N(x) \geqslant \alpha & \text{if } \alpha > 0, \\ cl(\text{supp } N) & \text{if } \alpha = 0, \end{cases}$$

and supp $N = \{x \in X | \mu_N(x) > 0\}$.

The fuzzy numbers most frequently used are the so-called trapezoidal fuzzy numbers. A fuzzy number N is trapezoidal if its membership function is defined as follows:

$$\mu_N(x) = \begin{cases} (x - a)\frac{1}{\alpha} + 1 & \text{if } a - \alpha \leqslant x \leqslant a, \\ 1 & \text{if } a < x < b, \\ (b - x)\frac{1}{\beta} + 1 & \text{if } b \leqslant x \leqslant b + \beta, \\ 0 & \text{otherwise.} \end{cases}$$

A trapezoidal fuzzy number may be seen as a fuzzy quantity "x is approximately in the interval $[a, b]$" and marked as $N = (a, b, \alpha, \beta)$ (Fig. 2).

In our system we introduce the trapezoidal fuzzy numbers to represent input and output variables in the fuzzy rules. Fuzzy rules take the form

if x is N_1 and y is N_2
or
if x is N_1 and y is N_2 then z is N_3 where
$N_i = (a_i, b_i, \alpha_i, \beta_i)$.

Furthermore we assume that the fuzzy inference process consist of the following steps (Yager, 1994):

- determination of the relevance or matching of each rule to the current input value;
- determination of the output of each rule as a fuzzy subset of the output space;
- aggregation of the individual rule outputs;
- selection of some action based upon the output set.

When introducing the aggregation operators we ask for the fulfillment of a set of conditions based on the three requirements: the indexing of the rules

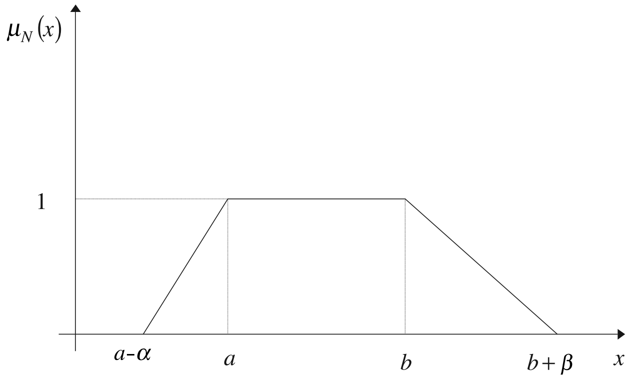

Fig. 2. Trapezoidal fuzzy number.

is unimportant, the association between individual rule output and total system output is positive, and non-firing rules play no role in the decision process.

The very general class of operators (MICA) was introduced by Yager (1994) and includes *t*-norms, *t*-conorms, averaging and compensatory operators.

The expert system knowledge base consists in a set of fuzzy and crisp rules located at two different hierarchical levels. The highest level of the system collects some intermediate outputs coming from the lowest level and aggregates them by means of a set of crisp rules. The lowest level prepares these intermediate outputs thanks to fuzzy inference processes that work on economic input variables.

As highlighted before, the use of an expert system has permitted to try a richer setting of rules for modelling the ECB behaviour, compared with what is allowed by the traditional computational techniques. The organization of the rules in different hierarchical levels and in different intra-level groups has allowed to manage the potentially contradictory structure of the ECB strategy.

As regards the fuzzy logic, its use has been suggested in order to have:

- more flexibility in defining the "alerting" values;
- more robustness in firing the rules;
- the usefulness of switching from fuzzy to crisp reasoning;
- a kind of reasoning closer to the human one.

2.2. The low-level fuzzy rules of the expert system

As we will explain in the next paragraph, the high-level crisp rules of the expert system match several macroeconomic scenarios with different profiles for the official interest rate path. The definition of the possible macroeconomic scenarios is done through three levels of resolution that bring to a classification into categories, groups and subgroups.

We start to explain how the low-level fuzzy rules support the classification into categories. For this classification the expert system has to assess whether from each of the two pillars come signals of inflationary–disinflationary risks. Starting from the first pillar, the expert system combines, thanks to the application of a fuzzy inference method on a set of rules, the input vector of monetary and credit indicators with an intermediate fuzzy output variable, which is called "*liquidity conditions in the Euro area*". Both the input variables and the intermediate output variables are linguistically defined. For the input variables they are simply linguistic quantifier like "low", "medium" or "high". For the intermediate output variables regarding the first pillar they are for example "relaxed" or "binding". A given vector of input values for the set of monetary and credit indicators considered can fire a certain number of predefined rules, which give the linguistic quantifiers of the intermediate output variable. For example, the combination of crisp input variables labelled "very low" fires the rule that produces the intermediate output labelled "very binding" (that means the "*liquidity conditions in the Euro area*" are "*very binding*"). The degree with which the consequence is fired depends on the membership degrees of the input variables and on the inference method applied. If we put the two opposite intermediate output linguistic quantifiers, i.e. "very relaxed" and "very binding", respectively on the right and left side of the domain at the intermediate output variable, we are able to give a logical meaning to the crisp value coming out from the defuzzification process. If the defuzzified value is above (below) a certain threshold the system will consider present risks of upward (downward) pressures on the prices in the medium term coming from the monetary and credit indicators. In particular, the intermediate output variable takes value one in presence of upward (downward) pressures and zero if they are not present. This is the information that is transferred to the high-level crisp rules.

This process is implemented in exactly the same way for the second pillar, of course with different variables and with different rules. The only difference is that the system, when assessing the presence of risks for the price stability coming from the second pillar, takes into consideration two different intermediate output variables. One regards the GDP indicators and the other regards real indicators, which are supposed to give more information about the level and persistency of economic growth (contraction). The system considers present risks only when these two intermediate output variables equal one.

The logic followed for the second and third level of resolution is exactly the same highlighted before. In particular, the second level states the contemporaneous presence of incipient pressures on the price and lagged effect. The third level states, instead, the contemporaneous presence of incipient pressure on the labour and on goods market.

The intermediate output variables transferred to the high-level crisp rules are the presence of incipient pressures in the job market and/or the goods market and the presence of risks of delayed effects. All together allow the classification into groups while the first two allow the classification into subgroups.

Therefore, at the low-level, fuzzy rules are organised in six different fuzzy inference systems, each of which gives as output a value for an intermediate output variable. The groups of indicators are: the monetary and credit indicators, the real GDP indicators, the level and persistence indicators; the labour market indicators; the goods market indicators; the lagged effects indicators.

Until now we have sketched the general computational framework in which the intermediate output variables are evaluated. We have not shown yet how the sets of fuzzy rules belonging to each of the six fuzzy inference systems have been designed and how the relations between these systems have been defined. In order to make this part of the expert fuzzy system clear we introduce three flowcharts. The first one refers to the first group (first pillar), the second to groups 2, 3, 4, 5 (second pillar) and the third one to group 6 (second pillar) (Figs. 3–5).

Each flowchart is the graphic version of a hierarchical reasoning in which the rightmost parts of the flowchart are reached only if the leftmost ones have been runned. The boxes on the extreme left refer to broad indicators of the current macroeconomic situation. If their values reflect possible risks for price stability in the medium term, the analysis has to go on at a further level involving other indicators that can make the picture clearer. In the first flowchart, the starting point of the analysis of the monetary and credit indicators is the comparison of the M3 growth with the announced reference value. If this first level of analysis is satisfied, i.e. M3 growth is above or below the reference value, the system takes into consideration the intensity and persistency of the deviation of M3 from the reference value. If also this second level of analysis is satisfied the system considers the evolution of the more liquid components of M3 (i.e. the narrow money aggregate M1) and of some counterparts of M3 (more details on the input variables will be given in the next paragraph). If also this level of analysis is satisfied, the system will consider present risks of upward (downward) pressures on prices coming from the first pillar of the Eurosystem monetary policy strategy. This cascade analysis starting from the left side of the flowchart and ending in its right side

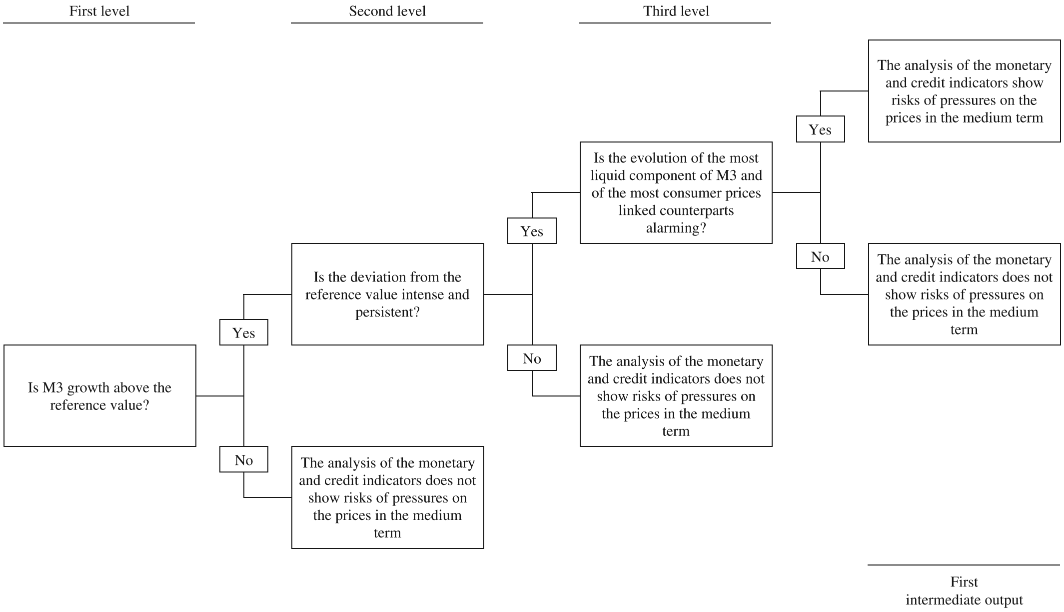

Fig. 3. Monetary and credit indicators analysis: First pillar.

originates several possible paths. This is because the input variables can trigger different levels of analysis with different levels of intensity. For example, the first level is satisfied with a value of the M3 growth equal to 4.6% but also with a value equal to 6%. Each of these paths can be represented by a rule that defines in which way the rightmost side of the flowchart is reached. These rules are the ones included in the first fuzzy inference system regarding the monetary and credit indicators. Being the rules fuzzy, they are defined using the linguistic quantifiers associated to each variable (“low”, “high”, etc.).

The second flowchart refers to the second pillar, i.e. the wide analysis of the perspective about the price evolution in the Euro area. The difference from the first flowchart is that here we obtain not only an intermediate output variable like in the previous case of monetary and credit indicators, but four intermediate outputs. In particular the intermediate outputs related to the gross domestic product, level and persistency of the growth, labour market and goods market indicators. In this case each rule of the four fuzzy inference systems linked to these indicators capture a way to reach not the rightmost side of the flowchart, but an intermediate right part of it, or passing from a certain level of the flowchart to the next. Anyway, the hierarchical structure introduced for the first flowchart is still valid in this second case. Even if it permits to evaluate more then one intermediate output variable, a certain level of analysis cannot be considered satisfied if the previous ones were not.

At the first level of analysis, we evaluate the GDP indicators. This first level of analysis is performed, whenever the input variables aggregation shows the presence of an economic growth (contraction) higher (lower) than the estimated potential one. The rules aggregated at this first level of analysis give the first intermediate output variable among the second pillar analysis.

Going right along the flowchart, the second level refers to the magnitude and persistency of the economic growth (contraction). At this particular step, the system analyses a set of real indicators, which are supposed to give more information about the current economic evolution. They include observed variables (industrial production, retail sales, export) as well as indicators of expectation estimated by private economic agents (industrial firms and consumer confidence indicators). At this

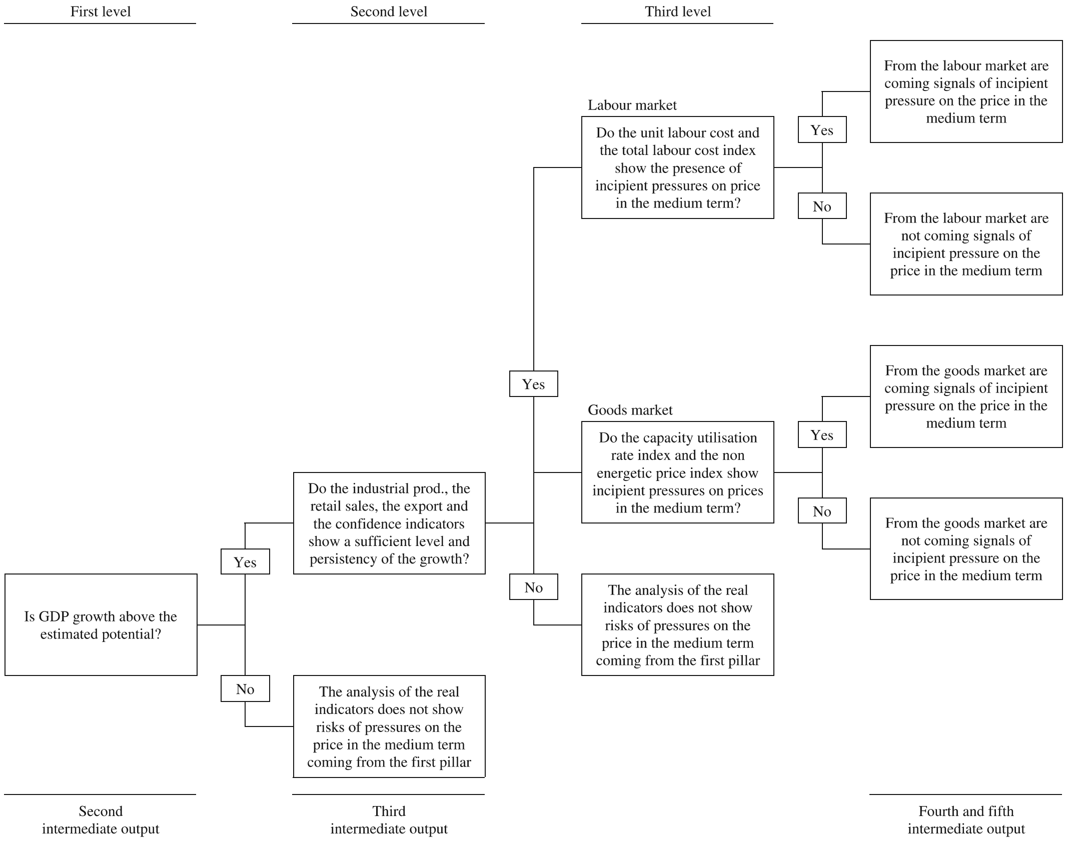

Fig. 4. Real indicators analysis: Second pillar.

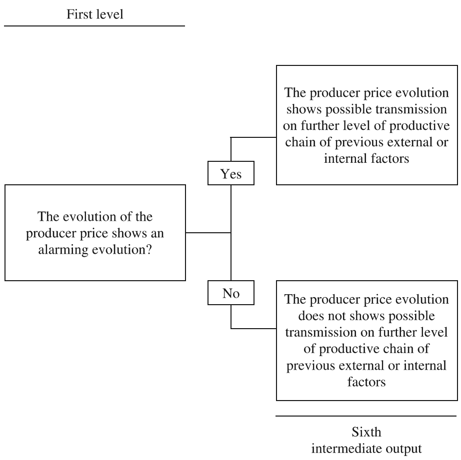

Fig. 5. Lagged effects indicators: Second pillar.

level of analysis, after comparing the input variables vectors with its set of rules, the second intermediate output of this second flowchart is given. If the system considers satisfied the requirements of level and persistency of the economic growth (contraction), the analysis moves to the next right level of the flowchart in which the incipient upward (downward) pressures on the prices in the labour market and in the goods market are considered. Both for the labour market and for the goods market are considered some indicators, which are supposed to signal possible pressures due to tensions arisen in the productive factor markets. They are the unit labour cost index and the total labour cost index for the labour market. The capacity utilisation rate index and the non-energetic raw materials price index for the goods market. After this final analysis, the last two intermediate output variables regarding the second pillar are available.

In conclusion, the economic analysis implemented by the system in this second module starts, like in the first one, with broad indicators. From each level to the following one (if the previous one is satisfied) it tries to make a more in-depth analysis of the inflationary (disinflationary) picture and the analysis ends when detecting incipient inflationary pressure.

In the third flowchart, we describe the module focusing on the analysis of the risk of delayed effects. The delayed effects are due to the transmission of current price movements to future price formation and salary setting. The particular high increase (decrease) of consumer prices can be caused either by the effect of previous exogenous shocks (oil price evolution or exchange rate evolution) or by the effect of previous pressures in the labour market and/or on the goods market, or by their joint presence. Anyway, when the effects of these factors on the evolution of the consumer prices are particular intense and persistent there is the risk of delayed effects, especially of price–salary tailspins. For this reason in the third flowchart the focus is on the evolution of production prices, which give information about the transmission on the further level of the productive chain of previous external or internal factors. In this third flowchart, which gives the last intermediate variable needed by the high-level crisp rules, the level of analysis is only one, because all the important information regarding the lagged effects are supposed to be given by the evolution of the production prices.

2.3. The high-level crisp rules of the expert-system

These rules are supposed to give the final output of the system, i.e. the forecasts about the ECB monetary policy intervention. They have been designed starting from the monetary policy strategy announced by the ECB.

The ECB has presented in a very clear way the main elements, which constitutes its strategy: the primary object of the price stability defined by a quantitative definition and the two pillars for the achievement of the primary object. Even if since its very beginning the ECB actions have been continuously and carefully evaluated, maybe its strategy has not been interpreted thoroughly.

These basic concepts constitute the knowledge base on which the crisp rules of the expert system have been designed. Each crisp rule combines a certain possible macroeconomic scenario with a path of monetary policy interventions. The classification of each macroeconomic scenario has been done, as highlighted in the previous paragraph, according to three levels of resolution:

- *first state*: the analysis of the monetary and real indicators is cross-checked in terms of pressure on the prices;
- *second state*: the contemporaneous presence of incipient pressures (coming from the labour market and/or from the goods market) and lagged effect on the prices;
- *third state*: the contemporaneous presence of incipient pressure on the labour and goods market.

Each level reflects a specific aspect of the analysis included in the ECB strategy carried out in order to make more understandable the current inflationary picture. Starting from the first level and going down to the third, the classification of the current macroeconomic scenario gets more and more accurate. The design of the possible macroeconomic scenarios (according to these three features) has been done considering all the possible combinations between the three levels.

After this combination exercise the number of possible scenarios has been fixed in 34, which should be a good compromise between the two opposite needs of mapping accuracy and model compactness.

Let us now introduce the particular features of the three levels of resolution. The first one is based on a six categories classification. The six categories are:

A. risk of upward pressures on prices in the medium term coming from both the two pillars of the Eurosystem monetary policy strategy;
B. risk of downward pressures on prices in the medium term coming from both the two pillars;

C. risk of upward pressures on prices in the medium term coming only from the second pillar;
D. risk of downward pressures on prices in the medium term coming only from the second pillar;
E. risk of upward pressures on prices in the medium term coming only from the first pillar;
F. risk of downward pressures on prices in the medium term coming only from the first pillar.

It is easy to detect how the above-introduced macroeconomic scenarios try to track the framework of the Eurosystem strategy. Indeed, being based on the two pillar concept, it cannot consider only the case in which the signals coming from them are converging. Just because the two pillars refer to two different vision of the economic structure, it is necessary that scenarios with non-perfectly converging signals be also considered. This may happen, for example, when neutral signals coming from the monetary and credit indicators coexist with signals of downward pressure on prices. We remember that a similar scenario has happened in April 1999 when the ECB has decided a reduction of the interest rate even in the absence of downward pressures coming from the first pillar.

The second level of resolution regards the division of each of the first four categories in four groups. This division, concerning particular aspects of the second pillar, has not been done for the last two categories in which there are not signals of pressure on prices. The four groups contain the following features:

Group 1: contemporary presence of incipient pressures on prices in the labour market and/or in the goods market and of risks of delayed effects due to previous exogenous shocks and/or transmission or previous pressures in the labour market and/or in the goods market;
Group 2: presence of incipient pressures on prices in the labour market and/or in the goods market;
Group 3: presence of risks of delayed effects due to previous exogenous shock and/or transmission or previous pressures in the labour market and/or in the goods market;
Group 4: absence of incipient pressures on the price in the labour market and/or in the goods market and of risks of delayed effects due to previous exogenous shock and/or transmission or previous pressures in the labour market and/or in the goods market.

This further classification of the macroeconomic scenarios try to catch the two main sources of risk, in terms of price stability, that the Eurosystem has to monitor continuously and counteract if necessary. The explicit reference to "incipient" pressures and "delayed effects" are the indirect recognition of the implication on monetary policy of the long time lags that characterise the transmission of the monetary impulse (European Central Bank, 2000).

The third and last resolution levels improve the accuracy of the classification introducing some subgroups. In each of the four groups belonging to each category the scenarios are defined depending on particular features. These are the presence of incipient pressure on prices both in the labour market and in the goods market; the presence of incipient pressure on the price only in the labour market; the presence of incipient pressure on prices only in the goods market.

2.4. The official interest rates paths assigned to different macroeconomic scenarios

In this paragraph we introduce the different official interest rates paths that the expert system matches with different macroeconomic scenarios. We adopt a dynamic setting: the decision by the ECB consists of an action to be taken (or not taken) immediately and a series of forecasted future interventions on a six months horizon. Assuming that the decisions are taken twice a quarter the series of forecasted interventions is constituted by three date. We think that, considering the high degree of uncertainty that surrounds the evolution of the economy, this reference period is a good compromise between usefulness and accuracy of the forecasts. Besides, this approach is consistent with the principles to which seems to be inspired the modern central banking (Blinder, 1998). According to these principles, the central banks reconsider quite frequently their policy stance defining a path of decisions to be taken in the future if the economic conjuncture evolves as planned.

Turning back to the official interest rate paths, in the previous paragraph we have not explicitly focused our analysis on the risk for price stability according to the prevailing scenario.

Starting from the first level of resolution, the first two categories introduced in the classification were the categories A and B. In particular, the A category refers to the upward risks and the B categories to the downwards risks.

Going down on the risk scale, we find the categories C and D. In these categories, the risk degree is lower than in the previous case because the cross-checking analysis is not satisfied. All the scenarios belonging to this category do not show any risk of pressure, either upward or downwards, coming from the monetary and credit indicators. The only risks for price stability come from the second pillar of the Eurosystem strategy, i.e. from perspectives about price evolution. These scenarios might seem at a first watch contradictory, but in reality they are not. They only record, as explained in the previous paragraph, the high degree of uncertainty surrounding the internal mechanism governing the economy. The absence of the satisfaction of the cross-checking analysis makes lower the risks for the price stability and increases the risks of taking wrong decisions or anyway decisions not perfectly consistent with the economic evolution.

However, even if these two categories are labelled (considering the same groups) as less risky compare with the first two, they are supposed to require a monetary policy intervention as well. This intervention will be, of course, less incisive in both the dimensions of the official interest rate path, i.e. the amount and the persistence of the intervention.

Going down on the risk scale we find the last two categories, E and F. In these two categories we have the same situation of categories C and D but inverted. The only risks for the price stability come from monetary and credit indicators. The reason why these two categories are considered less risky compared with the other, is the greater uncertainty surrounding the impact of periods of monetary expansion (restrictions) on the inflation rate.

Regarding the second level of resolution, we have shown in the previous paragraph how the classification in several groups has been done only for the first four categories. The two factors considered are the presence of incipient inflationary pressures in the labour market and/or in the goods market, and the risks of lagged effects, due to previous exogenous shocks (for example petrol or exchange rate shocks) or to previous pressure in the labour market and/or in the goods market.

It is intuitive that when these two sources of risk are jointly present, the inflationary or disinflationary picture is particularly alarming. The price stability, is not only put in danger by the incipient pressures observed in the productive factors market, but also by the current behaviour of the internal prices pushed up by lagged effects of internal and/or external factors. The first group of each category includes these kind of macroeconomic scenarios.

The second group contains, instead, all the macroeconomic scenarios, which show only pressures in the productive factors market, but do not show any risk of lagged effects. The third one includes those ones in which there are only risks of lagged effects. The second group, following the decreasing risk scale introduced with the first level of resolution, is considered more risky than the third one. This because lagged effects are more probable in a period in which are coming clear signals of tensions also from the productive market factors. Their priming is, instead, less probable in a macroeconomic context that does not present these features.

The presence of incipient pressures in the labour market and/or in the goods market is itself an alarming signal.

Finally, in the fourth group we have put the macroeconomic scenarios that do not present any of these features.

For what concerns the third level of resolution, we considered more risky the scenarios in which there is a contemporary presence of incipient pressures on the labour and goods market. When this condition is not satisfied we have given a higher risk grade to the scenarios showing pressures on the labour market. Indeed, the effects of the labour market evolution are usually more intense and persistent compared with those due to the goods market evolution.

We think that this classification based on three levels of resolution, allows not to forget the logical reference framework and in the same time to go in details for the individuation of the particular feature of each scenery.

The setting of each official interest rates path has been made bearing in mind the simple consideration that riskier scenarios have to be counteracted by the monetary authorities with more intensity and persistency (or better, using the ECB words with a "lower degree of lack of activism").

Therefore, inside each category these two features of the monetary policy are gradually weakened when the risk is lower. More specifically, the path of the categories C and D are those of the categories A and B reduced by 25 basis point. Anyway, the rule is not applied when the forecasted intervention amounts to 25 basis point. Being the scale of the possible intervention discrete, the application of the rule would imply the absence of any intervention.

The decrease of 25 basis point in the categories C and D is based on the higher risks of taking wrong decisions or decision non-perfectly consistent with the possible economic evolution. Because of this we think that in this situation the monetary policy has to inspire itself to lower activism.

Finally, for the categories E and F, we have decided for an inertial behaviour of the monetary authority in which the central bank monitors continuously the economic evolution but avoids interventions that could be inappropriate.

The Table 1 present the different path of the official interest rate combined with the different macroeconomic scenarios defined by the system. The table reflect the classification in categories, groups and scenarios introduced before. The letters a, b, c, d reflect the possible scenarios deriving from the third level of resolution. In particular:

(a) presence of incipient pressures on prices in the medium term coming from the labour market and from the goods market;
(b) presence of incipient pressures coming only from the labour market;
(c) presence of incipient pressures coming only from the goods market;
(d) absence of incipient pressures coming from the labour market or from the goods market.

3. Implementation and empirical results of the system

3.1. The training module

The system has been developed in the MATLAB programming environment. In particular we have used the MATLAB Fuzzy Logic Toolbox. The operator of the system can use, through a graphical interface, two different modules: the training and the recall module.

The first one allows for a self-tuning of the parameters that define the five membership functions attached to each input variable. This module has been created in order to:

- reduce as much as possible the subjectivity in fixing the membership function parameters;
- reduce the typical shortcoming of the expert system that is the "static" knowledge.

As regards the first task, it is worth to note that the final output of the system is particularly sensitive to the setting of these parameters. This because the fuzzy rules are fired only when all the linguistic quantifiers fixed in the rules syntax are activated by the input fuzzification. Only one mismatching does not permit the rule to be fired and give as output the most adequate fuzzy variable. Therefore, the setting of the membership parameters, which influences directly the linguistic quantifiers attached to each input variable, is one of the most crucial part of the fuzzy inference engine.

As regards the second issue, the self-tuning algorithm permits to give to the system some dynamic properties. The tuning of the parameters is done whenever the ECB takes a decision, which can be either an intervention or a non-intervention. The parameters are tuned according to the updated information and to the evolving judgement of the ECB about the economic structure of the Euro area.

Table 1
Macroeconomic scenarios and short-term official rate structure

Category	Group	Scenario	Official interest rate path			
			t	$t+1$	$t+2$	$t+3$
A	1	a	0.75	0.5	0.25	0.25
	1	b	0.5	0.5	0.25	0.25
	1	c	0.5	0.25	0.25	0.25
	2	a	0.5	0.25	0.25	0
	2	b	0.5	0.25	0.25	0
	2	c	0.5	0.25	0	0
	3	d	0.25	0.25	0.25	0
	4	d	0.25	0.25	0	0
B	1	a	−0.5	−0.5	−0.25	−0.25
	1	b	−0.5	−0.25	−0.25	−0.25
	1	c	−0.5	−0.25	−0.25	0
	2	a	−0.25	−0.25	−0.25	0
	2	b	−0.25	−0.25	−0.25	0
	2	c	−0.25	−0.25	0	0
	3	d	−0.25	−0.25	0	0
	4	d	−0.25	0	0	0
C	1	a	0.5	0.25	0.25	0.25
	1	b	0.25	0.25	0.25	0.25
	1	c	0.25	0.25	0.25	0.25
	2	a	0.25	0.25	0.25	0
	2	b	0.25	0.25	0.25	0
	2	c	0.25	0.25	0	0
	3	d	0.25	0.25	0	0
	4	d	0.25	0	0	0
D	1	a	−0.5	−0.25	−0.25	−0.25
	1	b	−0.25	−0.25	−0.25	−0.25
	1	c	−0.5	−0.25	−0.25	0
	2	a	−0.5	−0.25	0	0
	2	b	−0.5	−0.25	0	0
	2	c	−0.25	0	0	0
	3	d	−0.25	0	0	0
	4	d	−0.25	0	0	0
E	/	d	0	0	0	0
F	/	d	0	0	0	0

From the computational point of view, the self-tuning algorithm is similar to the gradient descent one used commonly in the backpropagation algorithm of the multilayer neural networks (Kasabov, 1996). In this case the algorithm try to minimise the total prediction error of the system, i.e. the sum of the prediction errors (in absolute values) at the times in which the ECB has taken a decision, changing the parameters of the membership functions. The main difference is that the expert system structure does not permit to define an analytical function which links changes in the total prediction error to changes in the parameters. For this reason we have been forced to use the numerical first partial derivatives of the total prediction error with respect to the parameters instead of the analytical ones. The algorithm, being iterative, needs to be run in a certain number of epochs fixed by the user. Anyway, also in this case can be applied all the methods which permit a better "movement" of the algorithm in the multidimensional error surface (Kasabov, 1996).

Therefore, the problem can be described analytically in the following way:

$$\min \sum_{j=1}^{n} \left| OI_j - FI_j \right| \rightarrow \min \text{TPE}$$

where OI_j and FI_j are, respectively, the observed intervention of the ECB and the forecasted one at time j and TPE the total prediction error. At each epochs the algorithm is run, the changes in the membership parameters are given by the following recursive formula:

$$\Delta P_{t+1}^{i,j} = \alpha \left(-\frac{\Delta \text{TPE}_t}{\Delta P_t^{i,j}} \right)$$

where $\alpha \in [0,1]$ is the learning rate and $P_t^{i,j}$ is the parameter of the ith membership functions of the jth variable at time t.

When the training is finished, the system is ready to be recalled in order to give a forecast about the official interest rate path.

3.2. The recall module

This module recall the system on a new set of input variables before that the ECB decision of intervention or non-intervention is known. It loads automatically the membership parameters when the training is finished. As highlighted in the previous paragraphs, the output is the forecast about the immediate action of the ECB and a series of future interventions that could be taken in the near future. Assuming that the intervention is taken twice a quarter, the forecast is about the next six months of monetary policy.

3.3. Empirical test

We have performed an empirical test simulating an application of the system along a fixed. The available time series, considering the introduced schedule of the decisions, included thirteen dates in which the ECB has taken a decision. Starting from the first one (in which the membership parameters has been fixed by us because no training was possible) we have run in each date the recall module and then the training one. After each recall the system has been trained with the time series virtually available at the moment.

The period considered goes from the first January 1999 to the 31 August 2000. As discussed just before, the thirteen dates have been fixed assuming an intervention calendar of the ECB with monetary policy decisions (that means also decisions of non-intervention) taken about twice in a quarter. This choice was suggested by modern central banking directions, which seem to give great importance to the ‘‘graduality’’ issue. The dates have been chosen respecting the official schedules for the ECB Governing Council meetings. In elaborating this independent intervention calendar we have tried also to give to the empirical test a meaning in terms of ability of the system in mimic the ‘‘timing function’’ of the ECB. The crisp input variables fed into the expert system at each date were the following ones:

Monetary and credit indicators

- MI1 last four months average of the three months centred moving average of the 12 months growing rate of M3 (percentage)
- MI2 number of months in which the MI1 deviation from the reference value has been observed
- MI3 last four months average of the 12 months growing rate of M1 (percentage)
- MI4 last four months average of the 12 months growing rate of the total credit to the Euro area resident (percentage)
- MI5 last four months average of the 12 months growing rate of the credit granted to the private sector of the Euro area (percentage)

Gross domestic real product indicators

- GI1 quarterly growth rate of the Euro area GDRP (percentage)
- GI2 annual growth rate of the Euro area GDRP (percentage)

Growth level and persistency indicators

- LPI1 annual growth rate of the industrial production excluding the construction sector (percentage)
- LPI2 annual growth rate of the retail sales (percentage)

LPI3 annual growth rate of the exports (decimal)
LPI4 quarterly change of the industrial firms confidence indicator (absolute)
LPI5 quarterly change of the consumers confidence indicator (absolute)

Labour market indicators
LMI1 annual growth rate of the unit labour cost index (percentage)
LMI2 annual growth rate of the total labour costs index (percentage)

Goods market indicators
GMI1 quarterly change of the capacity utilisation rate index (decimal)
GMI2 annual growing rate of the non-energetic raw materials price index (decimal)

Lagged effects indicators
LE1 annual growth rate of the producer prices (percentage)

The time series utilised are those in the statistical appendix of the ECB monthly bulletin. Apart from monetary and credit indicators in which we have considered some lagged variables, for all other input variables we have considered at each date the last available data.

In Tables 2 and 3 we present the results of a test for the years 1999 and 2000 at single date. In the first column we have the date in which the Governing Council has met. In second one we have the interventions calendar forecasted by the expert system and in the third one the one effectively set by the ECB. In Table 4 we report the short-term official Euro rate structure forecasted at each date by the system. The values in brackets show the value observed ex-post. In Table 5 we show for each date the value of the input variables.

Taking into account the complexity of the decision problem faced by the expert system, we think that its performances can be considered promising with respect to both the dimensions of the problem, i.e. the amount of the ECB intervention and the date in which it decides to intervene.

The system detects with a certain delay the start of the tightening period observed in November

Table 2
Comparison between the two interventions calendars for the 1999

Date of the ECB Governing Council meetings	Intervention calendar forecasted by the expert system	Effective intervention calendar set by the ECB
7 January	/	/
21 January	/	/
4 February	/	/
18 February	/	/
4 March	/	/
18 March	/	/
8 April	/	−0.5
22 April	/	/
6 May	/	/
20 May	/	/
2 June	/	/
17 June	/	/
1 July	/	/
15 July	/	/
29 July	/	/
26 August	/	/
9 September	/	/
23 September	/	/
7 October	/	/
21 October	/	/
4 November	/	+0.5
18 November	/	/
2 December	+0.25	/
16 December	/	/

Table 3
Comparison between the two interventions calendars for the 2000

Date of the ECB Governing Council meetings	Intervention calendar forecasted by the expert system	Effective intervention calendar set by the ECB
5 January	/	/
20 January	/	/
3 February	+0.50	+0.25
17 February	/	/
2 March	/	/
16 March	+0.50	+0.25
30 March	/	/
13 April	/	/
27 April	+0.50	+0.25
11 May	/	/
25 May	/	/
8 June	+0.50	+0.50
21 June	/	/
6 July	/	/
20 July	/	/
3 August	/	/
31 August	+0.50	+0.25

Table 4
Short term official rate structure forecasted by the expert system

Date (t)	Date ($t+1$)	Date ($t+2$)	Date ($t+3$)
7 January 1999	4 February 1999	18 March 1999	6 May 1999
0	0 (0)	0 (−0.5)	0 (0)
4 February 1999	18 March 1999	6 May 1999	17 June 1999
0	0 (−0.5)	0 (0)	0 (0)
18 March 1999	6 May 1999	17 June 1999	29 July 1999
0	0 (0)	0 (0)	0 (0)
6 May 1999	17 June 1999	29 July 1999	9 September 1999
0	0 (0)	0 (0)	0 (0)
17 June 1999	29 July 1999	9 September 1999	21 October 1999
0	0 (0)	0 (0)	0 (0.5)
29 July 1999	9 September 1999	21 October 1999	2 December 1999
0	0 (0)	0 (0.5)	0 (0)
9 September 1999	21 October 1999	2 December 1999	3 February 2000
0	0 (0.5)	0 (0)	0 (0.25)
21 October 1999	2 December 1999	3 February 2000	16 March 2000
0	0 (0)	0 (0.25)	0 (0.25)
2 December 1999	3 February 2000	16 March 2000	27 April 2000
0.25	0.25 (0.25)	0 (0.25)	0 (0.25)
3 February 2000	16 March 2000	27 April 2000	8 June 2000
0.50	0.25 (0.25)	0 (0.25)	0 (0.50)
16 March 2000	27 April 2000	8 June 2000	31 August
0.5	0.25 (0.25)	0.25 (0.50)	0.25 (0.25)
27 April 2000	8 June 2000	31 August	19 October 2000
0.5	0.25 (0.5)	0.25 (0.25)	0.25 (0.25)
8 June 2000	31 August	19 October 2000	30 November 2000
0.5	0.25 (0.25)	0.25 (0.25)	0.25 (/)
31 August	19 October 2000	30 November 2000	18 January 2001
0.5	0.25 (0.25)	0.25 (/)	0.25 (/)

and is not able to forecast the first intervention of the ECB in April 1999. In the rest of the test sample, the system shows promising performance. In particular, the analysis of the testing results and of Table 5 let us think that the aim of inserting in the expert system logic the "robustness" concept on which is based the Eurosystem monetary policy strategy has been reached in a quite good way.

From the date of May 1999 ahead, the expert system starts to point out a monetary expansion period. However, until the date of December 1999 this monetary expansion is not "cross-checked" by signals coming from the wide range of real indicators included in the second pillar. For this reason, and according to the Eurosystem monetary policy strategy, the expert system does not give any signal of intervention. In these months, from March 1999 to December 1999, a reaction function based only on monetary and credit indicators would have probably overvalued the risks of pressures on the prices in the medium term. From Table 5, in this period the values of the industrial production annual growth rate, of the export annual growth rate and of the quarterly change of the confidence indicators have not suggested the presence of risks of pressures on the prices in the medium term. Therefore, it seems that the system was able to reply in a satisfactory way the two-dimensional analysis present in the Eurosystem monetary policy strategy.

Anyway, it is worth to point out how in this period of transition the gross real domestic product indicators have shown, even if gradually, the possible shift to a tightening period. As said before, with a delay of about one month, the system has captured the start of the tightening period decided by the ECB in November 1999. The system seems to have correctly matched the increasing

Table 5
Input variables of the system

Date	Input variables																
	MI1	MI2	MI3	MI4	MI5	GI1	GI2	LPI1	LPI2	LPI3	LPI4	LPI5	LMI1	LMI2	GMI1	GMI2	LE1
Feb-99	4.55	3	8.73	7.00	9.10	0.40	2.70	2.80	4.00	−0.06	−1	1	−0.70	1.50	0.90	−0.17	−2.30
Mar-99	4.65	3	10.15	7.20	9.48	0.30	2.40	−0.50	4.00	0.00	0	1	−0.60	1.50	0.90	−0.16	−2.40
May-99	5.08	3	12.05	7.43	9.73	0.30	2.30	−0.20	1.90	−0.07	−3	0	−0.60	1.50	−0.90	−0.13	−2.70
Jun-99	5.25	3	12.63	7.50	9.90	0.30	2.30	−0.30	5.50	−0.06	−2	−3	−0.60	1.50	−0.90	−0.08	−2.30
Jul-99	5.38	3	11.98	7.53	9.90	0.70	1.80	−0.70	1.60	−0.03	−11	−4	−0.40	1.50	0.00	−0.04	−1.60
Sep-99	5.43	3	12.30	7.55	10.43	0.70	1.80	0.80	1.00	0.02	4	0	−0.35	1.85	0.00	0.00	−1.00
Oct-99	5.53	3	12.63	7.68	10.63	0.60	1.50	0.10	3.50	0.01	4	0	−0.20	1.85	−0.20	−0.01	−0.40
Dec-99	5.73	3	13.18	7.75	10.53	0.60	1.60	1.40	1.80	0.05	4	1	0.20	2.15	−0.20	0.12	1.30
Feb-00	5.85	3	11.90	8.20	10.63	1.00	2.20	2.80	2.70	0.11	0	−1	1.70	2.10	0.20	0.19	3.00
Mar-00	5.85	3	11.03	8.08	10.38	1.00	2.20	4.50	2.40	0.12	4	1	1.20	2.20	0.20	0.19	4.10
Apr-00	5.85	3	10.45	8.08	10.38	0.80	3.00	3.60	2.60	0.21	4	1	1.10	2.20	1.00	0.20	5.70
Jun-00	6.00	3	10.30	7.80	10.58	0.80	3.10	5.00	1.80	0.18	4	1	0.65	2.30	1.00	0.23	6.30
Aug-00	6.20	3	9.15	7.58	10.85	0.90	3.40	7.70	3.70	0.35	8	−1	0.35	3.00	0.80	0.14	6.60

path of the industrial production and of the two confidence indicators. The industrial production value has changed, indeed, from the (0.1) of October to the (1.4) of December.

From that date ahead, the expert system gives good performance regarding both the dimensions of the problem, but with a tendency in overestimating the amount of the intervention. In the date of February 2000 the system has correctly detected the strengthening of the industrial production, of the retail sales and of the exports that have changed respectively from (1.4) to (2.8), from (1.8) to (2.7) and from (0.05) to (0.11). In this date the expert system has given signals of risks coming from the growth level and persistence indicators even if the confidence indicators were showing a slowing down. This is due to the presence of low-level fuzzy rules that take into account these cases in which some indicators, even if at medium values, are not at the high-level of the others.

From March 2000 onwards the expert system has correctly detected the macroeconomic picture. Indeed, since that date the macroeconomic indicators have shown a period of strong economic growth, the presence of incipient pressures in the goods market, together with risks of delayed effects due to the delayed effects of exogenous shocks, i.e. oil price increases and depreciation of the Euro against the US dollar. Looking to the input variables, it is apparent in this period the continuos increase in producer prices (signal of the transmission to the consumer prices of the previous shocks), in the capacity utilisation rate index and in the prices of the non-energetic raw materials.

4. Conclusions

Trying to model a complex decisional process, like that of a central bank, using tools coming from Artificial Intelligence is a very difficult task. A monetary policy strategy requires a set of defined economic relationships enshrined in the economic model adopted by a central bank, but also an additional important ingredient: judgement, that is to say, discretion or lack of automatism (Solans, 2000). Combining efficiently both of those components in only one system makes the task mentioned before a very challenging one. For this reason, the expert system introduced in this work has not the ambition to guide the choices of all those people influenced by the monetary policy decisions of the ECB. In our work, we want to contribute to the reduction of the "communication gap" between the ECB and its watchers. The decision support system can be used as a component of a more structured and wider decision process.

Anyway, in a new economic structure, like the Euro area, the signals coming from monetary and real indicators are difficult to interpret with the model used until now. We think that the intrinsic nature and complexity of the modern monetary policy needs the introduction of models based on fuzzy systems. With our models we can capture this kind of complexity, giving to the interested financial operators a realistic answer of the monetary policy of the ECB in almost any possible economic situations of the Euro area.

Possible improvements may come from introducing in the system at the high-level some additional behavioural rules (Blinder, 1997, 1998) in order to capture the "lack of activism" approach on which the ECB monetary policy strategy is inspired (Solans, 2000). Our model also can be improved including some exogenous variables, like for example, the behaviour of the Fed, the Central Bank of the United States. This can be useful in particular situations—like the turbulent environment after the 11 September—in which the monetary decision of the ECB can be based not only on the values of the economic variables.

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